CBCS SCHEME

USN 18MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$ (08 Marks)

b. Expression the complex number $(2+3i)+\frac{1}{1-i}$ in the form a + ib. (06 Marks)

c. Find the modulus and amplitude of the complex number $1-\cos\alpha + i\sin\alpha$. (06 Marks)

OR

2 a. If $\vec{A} = i+2j-3k$, $\vec{B} = 3i-j+2k$ show that $\vec{A}+\vec{B}$ is perpendicular to $\vec{A}-\vec{B}$. Also find the angle between $2\vec{A}+3\vec{B}$ and $\vec{A}+2\vec{B}$. (08 Marks)

b. Show that the vectors $i-2j+3k$, $2i+j+k$, $3i+4j-k$ are coplanar. (06 Marks)

c. Find the sine of the angle between $\vec{A} = 4i-j+3k$ and $\vec{B} = -2i+j-2k$. (06 Marks)

Module-2

3 a. Obtain the Maclaurin's series expansion of sin x upto term containing x^4. (08 Marks)

b. If $u = \sin^{-1}\left[\frac{x^2+y^2}{x-y}\right]$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \tan u$. (06 Marks)

c. If $u = f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$. (06 Marks)

OR

4 a. Prove that $\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots\dots\dots$ by using Maclaurin's series. (08 Marks)

b. If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$. (06 Marks)

c. If $z = e^{ax+by} f(ax-by)$ then show that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y} = 2abz$. (06 Marks)

Module-3

5 a. Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point (2, –1, 2) (08 Marks)

b. Find the unit vector normal to the surface $x^2y+2xz = 4$ at (2, –2, 3). (06 Marks)

c. Show that the vector $(-x^2+yz)i+(4y-z^2x)j+(2xz-4z)k$ is solenoidal. (06 Marks)

OR

6 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$. **(08 Marks)**

b. Find the values of a, b, c such that $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational. **(06 Marks)**

c. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(06 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$, $n > 0$. **(08 Marks)**

b. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1 - x^2}} dx$ **(06 Marks)**

c. Evaluate $\iint xy(x + y) dx\, dy$ over the area between $y = x^2$ and $y = x$. **(06 Marks)**

OR

8 a. Obtain the reduction formula for

$\int_0^{\pi/2} \sin^n x \, dx$, $n > 0$. **(08 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1 - x^2)^{7/2}} dx$ **(06 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz\, dy\, dx$ **(06 Marks)**

Module-5

9 a. Solve $y(\log y)dx + (x - \log y)dy = 0$ **(08 Marks)**

b. Solve $x.\frac{dy}{dx} + y = x^3 y^6$ **(06 Marks)**

c. Solve $(xy^2 - e^{1/x^3})dx - x^2 y\, dy = 0$ **(06 Marks)**

OR

10 a. Solve $(5x^4 + 3x^2y^2 - 2xy^3)\, dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ **(08 Marks)**

b. Solve $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$ **(06 Marks)**

c. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)\, dy = 0$ **(06 Marks)**

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